**Statistical Inference**

###### Methods of Point Estimation

We have discussed different properties of good point estimators, viz. unbiasness, minimum variance, consistency and efficiency. Our endeavor will be to search for estimators, which will have as many of these optimum properties as possible. There are different methods of point estimation, which are expected to yieldestimators enjoying some of these important properties. Some of these methods are

* Method of maximum likelihood
* Method of moments
* Method of least squares
* Method of minimum variance
* Method of minimum
* Method of inverse probability

Out of all the methods of point estimation, method of maximum likelihood is the best. It is the easiest method and at the same time the maximum likelihood estimator possesses several satisfactory properties.

**Maximum Likelihood Estimator**

* For the sample observations the joint p.d.f. (for continuous distribution)/p.m.f. (for discrete distribution) can be written as

where

* Clearly, is a function of only, since are known being sample observations. Consequently, the joint p.d.f. / p.m.f. is called the likelihood function of and is denoted by. For an independent and identically distributed sample, this joint density function is
* The particular value of which maximizes is considered as an estimate of . Thus, if is the maximum likelihood estimate of, then by definition, .
* Often instead of maximizing, we maximize logarithm of. The base of the logarithm may be or 10. Since is a monotonically increasing function of, the value of that maximizes also maximizes , i.e.

**Procedure for obtaining Maximum Likelihood Estimator (MLE)**

1. Obtain maximum likelihood function as

,

1. Determine the value of by solving , or

1. Verify that or is negative for the value of as obtained in step 2.

* **Obtain the MLE of the binomial parameter in .**

is considered as the outcome of Bernollian trials with the outcomes , where

And, and Now, the maximum likelihood function,

,

where is the total number of success and each sample observation (trial) could be either a success with probability or a failure with probability .

So,

, since

So, maximizes the likelihood function and hence is the MLE of .

* **In random sampling from a normal population , find the maximum likelihood estimators for**

1. **when is known,**
2. **when is known, and**
3. **and, when both and are unknown.**

or,

1. When is known,

So, MLE for is the sample mean .

1. When is known,

(say)

Again,

Hence, MLE of is .

1. When both and are unknown,

(1)

(2)

The solution of the simultaneous equations (1) and (2) will give the MLE of and , and it is found that

, and

Now, considering that and are given, we have

And, , which is positive

Clearly, the determinant of the hessian matrix is positive and the diagonal elements are negative. Therefore, using second partial derivative test, we conclude that , maximizes . Hence, and are respectively the MLE’s of and respectively.

* ***The probability distribution of Laplace distribution is as follows: ,* . *Determine the MLE of .***

*In order to find the value of which maximizes , we might consider arranging , ,…, in ascending order of magnitude.*

*, ,…, below*

*, ,…, above , with unknown*

*Thus, we would have,*

*…*

*…*

*or,*

That is, the MLE of is the median, i.e. .

But . So we have no guarantee that maximizes the log likelihood function.

**Properties of Maximum Likelihood Estimators**

1. In general, the MLE of a parameter is a consistent estimator.
2. In general, MLE is asymptotically normally distributed as .
3. In general, MLE is efficient. The variance of MLE is usually equal to the Cramer-Rao lower bound.
4. MLE possesses the property of invariance. If  is the MLE of , and if is a continuous monotone function of , then the MLE of is .
5. If a sufficient statistic exists, MLE is that statistic or a function of it.
6. If there exists a minimum variance unbiased estimator of a parameter *θ* , then MLE coincides with the same.
7. The only drawback of MLE is that it may not be an unbiased estimator. For example, for normal population MLE of variance is not an unbiased estimator.

**Exercise 1:** Let be exponentially distributed with parameter. Find the maximum likelihood estimator of .

**Exercise 2:** Let be uniformly distributed with parameter and , and the p.d.f. of the uniform distribution is

, .

Find the maximum likelihood estimators of and .

**Method of Moments**

This is one of the classical methods of point estimation. In this method, the raw sample moments are equated with corresponding raw moments of the population and from this the parameters of the population are estimated. Sample and population raw moments of *r*-th order are, respectively, given by

and

The number of moments to be equated depends upon the number of unknown parameters.

* **Using method of moments estimate the parameters of the following probability distributions:**

****

i) and

ii) and

and

Therefore**,** and

So,

iii) and

Therefore,

* **Using method of moments estimate the parameters of a Gamma population. The probability density function of the Gamma distribution is given below:**

**, for and**

**and are the shape and scales parameters of the Gamma distribution. The mean and variance the Gamma distribution are and respectively.**

and

and

Thus, equating the corresponding moments of sample and population we get

(1)

(2)

Solving the above two equations we get,

and

**Weakness of Methods of Moments**

1. Moments of an order may not exist.
2. Not efficient as higher order moments are subject to high fluctuations.

**Method of Least Squares**

This method of estimation is used mainly for fitting a curve of the following form to the observed sample observations:

where are unknown parameters

In this method, we select the parameter estimates such that the ***sum of squares of the residuals*** becomes **minimum**, i.e.

becomes minimum

**Note that**

* Least square estimators do not have any optimum property, even asymptotically. However, in linear estimation case, this method provides good estimator in small samples.
* When is a linear function of parameters the least squares estimators will be minimum variance unbiased (MVU) estimators.